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# 澳門四高校聯合入學考試 (語言科及數學科)

# Joint Admission Examination for Macao Four Higher Education Institutions (Languages and Mathematics)

## 2025 試題及參考答案
## 2025 Examination Paper and Suggested Answer

## 數學正卷 Mathematics Standard Paper

**第一部份 選擇題。請選出每題之最佳答案。**

1. 設集合 $A=\{x:x^2+3x-4\geq 0\}$，$B=\{-4,-2,0,3\}$，則 $A\cap B=(\quad)$。

   A. $\{-4,3\}$　　B. $\{-4,-2\}$　　C. $\{-2,3\}$　　D. $\{0,3\}$　　E. $\{-2,0\}$

2. 若 $\frac{1}{\alpha}$ 和 $\frac{1}{\beta}$ 是方程 $2x^2+2x-1=0$ 的根，則 $2^{\alpha+1}\times 2^{\beta+1}=(\quad)$。

   A. 1　　B. 2　　C. 4　　D. 16　　E. $\frac{1}{16}$

3. 若一個正圓柱體的底半徑增加 30%，而其高度同時減少 30%，其體積將 ( )。

   A. 增加 18.3%　　B. 增加 9%　　C. 減少 9%

   D. 減少 6%　　E. 維持不變

4. $\dfrac{m^{\frac{3}{2}}-m^{-\frac{1}{2}}}{m^{\frac{1}{2}}+m^{-\frac{1}{2}}}=(\quad)$。

   A. $m$　　B. $m+1$　　C. $m-1$　　D. $m^2+1$　　E. $m^2-1$

5. 若 $2^p=5$ 及 $2^q=7$，則 $\log_2 0.7=(\quad)$。

   A. $q+p-1$　　B. $2q-2p$　　C. $q-p+1$

   D. $q-p-1$　　E. 以上皆非

6. 不等式 $|x(x-5)|<6$ 的解集為 ( )。

   A. $\{-1<x<6\}$　　B. $\{-2<x<1\}$　　C. $\{-1<x\leq 1\}\cup\{4\leq x\leq 5\}$

   D. $\{x\leq -1\}\cup\{x\geq 6\}$　　E. $\{-1<x<2\}\cup\{3<x<6\}$

7. 四個女孩和三個男孩排成一行。若不允許男孩連排，則可能的排列有 ( ) 種。

   A. 144　　B. 288　　C. 1440　　D. 2880　　E. 5760

8. 若六個數 $x+2, x+3, x+4, x-4, x-5, x-6$ 的中位數是 8，則這六個數的平均值是 ( )。

   A. 3　　B. 7　　C. 8　　D. 9　　E. $x$

9. $(x+\frac{y^3}{x^2})(x+y)^8$ 的展開式中 $x^4y^5$ 的係數為 (　　)。

A. 65　　B. 84　　C. 94　　D. 127　　E. 176

10. 已知圓 $x^2-4x+y^2=0$，過點 $(1,1)$ 的直線被該圓所截得的弦的長度最小值是 (　　)。

A. 1　　B. $2\sqrt{3}$　　C. 2　　D. $\sqrt{5}$　　E. $2\sqrt{2}$

11. 已知 $A$ 為拋物線 $\mathcal{C}: x^2=-py\ (p>0)$ 上的一點。點 $A$ 到 $\mathcal{C}$ 的焦點的距離為 15，到 $x$ 軸的距離為 7，則 $p=$ (　　)。

A. 14　　B. 15　　C. 16　　D. 28　　E. 32

12. 在射擊遊戲中，約翰和安娜每次獨立射擊成功擊中目標的概率分別為 $\frac{1}{3}$ 和 $\frac{2}{3}$。每人射擊三次的情況下，約翰和安娜成功擊中目標次數總和為 4 的概率為 (　　)。

A. $\frac{16}{27}$　　B. $\frac{64}{81}$　　C. $\frac{25}{81}$　　D. $\frac{58}{243}$　　E. $\frac{34}{243}$

13. 已知等差數列 $\{a_n\}_{n=1}^{\infty}$ 的前 $n$ 項和為 $S_n$，且 $2S_3^2=3S_2S_4$，$a_1=4$，則 $a_7=$ (　　)。

A. $-14$　　B. $-8$　　C. $-2$　　D. 10　　E. 18

14. 若 $x$，$y$ 滿足約束條件 $\begin{cases} 3x+4y\le 7 \\ x-2y\ge -1 \\ y\ge -1 \end{cases}$，則 $z=3x+y$ 的最大值是 (　　)。

A. 4　　B. 6　　C. 7　　D. 10　　E. 11

15. 定義在 $\mathbb{R}$ 上的函數 $f(x)$ 滿足 $f(x)=f(x+2)$。當 $x\in[4,6]$ 時，$f(x)=1+|x-5|$，則下列不等式不正確的是 (　　)。

A. $f(\sin\frac{\pi}{6})>f(\cos\frac{\pi}{6})$　　B. $f(\sin\frac{\pi}{3})>f(\cos\frac{\pi}{3})$　　C. $f(\cos\pi)<f(\sin\pi)$

D. $f(\sin\frac{2\pi}{3})<f(\cos\frac{2\pi}{3})$　　E. $f(\sin\frac{\pi}{2})<f(\cos\frac{\pi}{2})$

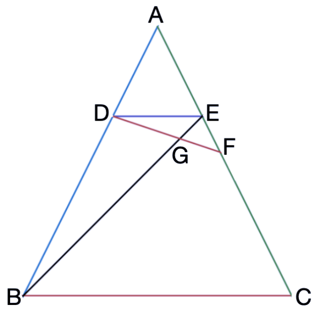
**第二部份** **解答題。**

1. 設數列 $\{a_n\}_{n=1}^{\infty}$ 的前 $n$ 項和 $S_n = n^2 + 2n$。公比為正數的等比數列 $\{b_n\}_{n=1}^{\infty}$ 中，$b_1 = 2$ 且 $b_3 = 2a_4$。
   (a) 求數列 $\{a_n\}_{n=1}^{\infty}$ 和 $\{b_n\}_{n=1}^{\infty}$ 的通項。 (4 分)
   (b) 設 $c_n = a_n b_n$。求數列 $\{c_n\}_{n=1}^{\infty}$ 的前 $n$ 項和 $T_n$。 (4 分)

2. 設 $f(x) = 8x^4 + ax^3 + bx^2 + cx + 9$，其中 $a$、$b$ 和 $c$ 為常數。已知當 $f(x)$ 除以 $x+1$ 時餘數為 $-10$ 及 $f(x) \equiv (px^2 - 3x + 3)(2x^2 + qx + r)$，其中 $p$、$q$ 和 $r$ 是常數。
   (a) 求 $p$、$q$ 和 $r$ 的值。 (3 分)
   (b) 求方程 $f(x) = 0$ 的實數根。 (5 分)

3. 在 $\triangle ABC$ 中，$\sin(A+B) = 6\sin^2\dfrac{C}{2}$。
   (a) 求 $\cos C$。 （4 分）
   (b) 若 $\angle A = 45^\circ$，求 $\sin 2B$。 （4 分）

4. 在 $\triangle ABC$ 中，$AB = AC$，$AD = AE$，點 $F$ 在邊 $AC$ 上，$DF$ 與 $BE$ 相交於點 $G$，且 $\angle AFD = \angle DEB$。
   (a) 證明 $\triangle DEG \sim \triangle DFE$。 （3 分）
   (b) 證明 $\triangle DEF \sim \triangle BDE$。 （3 分）
   (c) 證明 $DG \cdot DF = DB \cdot EF$。 （2 分）




5. 已知點 $A(-2\sqrt{2}, 0)$ 和 $B(2\sqrt{2}, 0)$，動點 $M(x, y)$ 滿足直線 $AM$ 與 $BM$ 的斜率之乘積為 $-\dfrac{1}{2}$。設 $M$ 的軌跡為曲線 $\mathcal{C}$。
   (a) 求 $\mathcal{C}$ 的方程。 (4 分)
   (b) 直線 $\ell : y = kx + b$ $(k, b \neq 0)$ 與曲線 $\mathcal{C}$ 相交於 $P$ 和 $Q$ 兩點。線段 $PQ$ 的中點為 $D$，坐標原點為 $O$。求直線 $OD$ 的斜率（用 $k$ 表達）。 (4 分)

# 參考答案

**<u>第一部份</u> 選擇題。**

| 題目編號 | 最佳答案 |
| --- | --- |
| 1 | A |
| 2 | D |
| 3 | A |
| 4 | C |
| 5 | D |
| 6 | E |
| 7 | C |
| 8 | C |
| 9 | B |
| 10 | E |
| 11 | E |
| 12 | D |
| 13 | B |
| 14 | D |
| 15 | B |

**第二部份** **解答題。**

1. (a) 當 $n=1$ 時，$a_1=3$。當 $n\geq 2$ 時，$a_n=S_n-S_{n-1}=n^2+2n-(n-1)^2-2(n-1)=2n+1$。因此 $n\geq 1$，$a_n=2n+1$。因為 $a_4=9$，所以 $b_3=2a_4=18$。設等比數列 $\{b_n\}_{n=1}^{\infty}$ 的公比為 $q(>0)$，則 $b_3=b_1q^2$，由於 $b_1=2$，於是 $18=2q^2$，解得 $q=3$。因此 $b_n=2\cdot 3^{n-1}$。

   (b) 由 $c_n=a_nb_n$ 和 $T_n=c_1+c_2+\cdots+c_n=3\cdot 2+5\cdot 6+7\cdot 18+\cdots+(2n+1)\cdot 2\cdot 3^{n-1}$ 知 $3T_n=3\cdot 6+5\cdot 18+\cdots+(2n-1)\cdot 2\cdot 3^{n-1}+(2n+1)\cdot 2\cdot 3^n$，於是 $2T_n=3T_n-T_n=-3\cdot 2-2\cdot 6-2\cdot 18+\cdots-2\cdot 2\cdot 3^{n-1}+(2n+1)\cdot 2\cdot 3^n=-6-2(6+18+\cdots+2\cdot 3^{n-1})+(2n+1)\cdot 2\cdot 3^n=-6-2\cdot\dfrac{6(1-3^{n-1})}{1-3}+(2n+1)\cdot 2\cdot 3^n=4n\cdot 3^n$，因此 $T_n=2n\cdot 3^n$。

2. (a) 考慮 $f(x)$ 中 $x^4$ 的係數及常數項，容易知道 $2p=8$ 及 $3r=9$，從而 $p=4$ 及 $r=3$，於是 $f(x)=(4x^2-3x+3)(2x^2+qx+3)$。因為當 $f(x)$ 除以 $x+1$ 時餘數為 $-10$，所以 $f(-1)=-10$，即 $10(5-q)=-10$，解得 $q=6$。

   (b) 由 $f(x)=(4x^2-3x+3)(2x^2+6x+3)=0$ 知，$4x^2-3x+3=0$ 或 $2x^2+6x+3=0$。對 $4x^2-3x+3=0$，由於判別式 $\Delta=(-3)^2-4(4)(3)=-39<0$，方程無實數根。對 $2x^2+6x+3=0$，判別式 $\Delta=6^2-4(2)(3)=12>0$，方程有兩個（不相等的）實數根，由求根公式知方程兩實數根分別為 $x_1=\dfrac{-3+\sqrt{3}}{2}$ 和 $x_2=\dfrac{-3-\sqrt{3}}{2}$。

3. (a) 因為 $\sin C=\sin(A+B)=6\sin^2\frac{C}{2}$，所以 $\sin C=3(1-\cos C)$。又由於 $\sin^2 C+\cos^2 C=1$，於是 $9(1-\cos C)^2+\cos^2 C=1$，整理得 $5\cos^2 C-9\cos C+4=(5\cos C-4)(\cos C-1)=0$。因為 $C$ 為三角形的內角，所以 $\cos C-1\neq 0$。於是 $5\cos C-4=0$，從而 $\cos C=\dfrac{4}{5}$。

   (b) 因為 $\angle A=45^\circ$，所以 $\sin 2A=1$，$\cos 2A=0$。於是 $\sin(2B)=-\sin(2A+2C)=-\sin 2A\cos 2C-\cos 2A\sin 2C=-\cos 2C=1-2\cos^2 C=1-(\dfrac{4}{5})^2=-\dfrac{7}{25}$。

4. (a) 在 $\triangle DEG$ 和 $\triangle DFE$ 中，$\angle DEG=\angle DFE$（已知），$\angle EDG=\angle FDE$（同角），因此 $\triangle DEG\sim\triangle DFE$。(本題有多種解法，此處不逐一列舉。)

   (b) 因為 $AD=AE$，所以 $\angle ADE=\angle AED$，從而 $\angle DEF=\angle BDE$。又因為 $\angle AFD=\angle DEB$，所以 $\angle EFD=\angle DEB$。因此，$\triangle DEF\sim\triangle BDE$。(本題有多種解法，此處不逐一列舉。)

(c) 因為 $\triangle DEG \sim \triangle DFE$，所以 $\frac{DE}{DF} = \frac{DG}{DE}$，於是 $DE^2 = DG \cdot DF$。又因為 $\triangle DEF \sim \triangle BDE$，所以 $\frac{DE}{BD} = \frac{EF}{DE}$，於是 $DE^2 = DB \cdot EF$。因此 $DG \cdot DF = DB \cdot EF$。

5. (a) 若設 $M(x, y)$，則 $AM$ 的斜率 $k_{AM} = \dfrac{y}{x+2\sqrt{2}}$，$BM$ 的斜率 $k_{BM} = \dfrac{y}{x-2\sqrt{2}}$。由題意知，$k_{AM} \times k_{BM} = -\dfrac{1}{2}$，即 $\dfrac{y}{x+2\sqrt{2}} \times \dfrac{y}{x-2\sqrt{2}} = -\dfrac{1}{2}$，化簡得曲線 $\mathcal{C}$ 的方程為：$\dfrac{x^2}{8} + \dfrac{y^2}{4} = 1$。

(b) 聯立直線 $\ell$ 和曲線 $\mathcal{C}$ 的方程 $\begin{cases} y = kx + b \\ \dfrac{x^2}{8} + \dfrac{y^2}{4} = 1 \end{cases}$，消 $y$ 得 $\dfrac{x^2}{8} + \dfrac{(kx+b)^2}{4} = 1$，整理得 $(2k^2+1)x^2 + 4kbx + 2b^2 - 8 = 0$。由韋達定理及中點坐標公式得 $D$ 點的 $x-$ 坐標為 $x_D = -\dfrac{2kb}{2k^2+1}$，從而 $D$ 點的 $y-$ 坐標為 $y_D = \dfrac{b}{2k^2+1}$。因此，直線 $OD$ 的斜率 $k_{OD} = \dfrac{y_D}{x_D} = -\dfrac{1}{2k}$。

**<u>Part I</u> Multiple choice questions. Choose the *best answer* for each question.**

1. Let sets $A = \{x : x^2 + 3x - 4 \geq 0\}$ and $B = \{-4, -2, 0, 3\}$. Then $A \cap B =$ (    ).

   A. $\{-4, 3\}$ B. $\{-4, -2\}$ C. $\{-2, 3\}$ D. $\{0, 3\}$ E. $\{-2, 0\}$

2. If $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are roots of the equation $2x^2 + 2x - 1 = 0$, then $2^{\alpha+1} \times 2^{\beta+1} =$ (    ).

   A. 1 B. 2 C. 4 D. 16 E. $\frac{1}{16}$

3. If the base radius of a cylinder increases by 30%, while its height simultaneously decreases by 30%, then the volume of the cylinder will (    ).

   A. increase by 18.3% B. increase by 9% C. decrease by 9%

   D. decrease by 6% E. remain unchanged

4. $\dfrac{m^{\frac{3}{2}} - m^{-\frac{1}{2}}}{m^{\frac{1}{2}} + m^{-\frac{1}{2}}} =$ (    ).

   A. $m$ B. $m + 1$ C. $m - 1$ D. $m^2 + 1$ E. $m^2 - 1$

5. If $2^p = 5$ and $2^q = 7$, then $\log_2 0.7 =$ (    ).

   A. $q + p - 1$ B. $2q - 2p$ C. $q - p + 1$

   D. $q - p - 1$ E. None of the above

6. The set of solutions of the inequality $|x(x - 5)| < 6$ is (    ).

   A. $\{-1 < x < 6\}$ B. $\{-2 < x < 1\}$ C. $\{-1 < x \leq 1\} \cup \{4 \leq x \leq 5\}$

   D. $\{x \leq -1\} \cup \{x \geq 6\}$ E. $\{-1 < x < 2\} \cup \{3 < x < 6\}$

7. Four girls and three boys are to be arranged in a single line. If boys are not allowed to stand next to each other, then there are (    ) different possible arrangements.

   A. 144 B. 288 C. 1440 D. 2880 E. 5760

8. If the median of six numbers $x + 2, x + 3, x + 4, x - 4, x - 5, x - 6$ is 8, then the mean of these six numbers is (    ).

   A. 3 B. 7 C. 8 D. 9 E. $x$

9. The coefficient of the term $x^4y^5$ in the expansion of $(x+\frac{y^3}{x^2})(x+y)^8$ is ( ).

A. 65 B. 84 C. 94 D. 127 E. 176

10. Given the circle $x^2-4x+y^2=0$, the minimum length of the chord intercepted by the circle from a line passing through the point $(1,1)$ is ( ).

A. 1 B. $2\sqrt{3}$ C. 2 D. $\sqrt{5}$ E. $2\sqrt{2}$

11. Given that $A$ is a point on the parabola $C: x^2=-py\ (p>0)$, the distance from the point $A$ to the focus of $C$ is 15, and the distance from $A$ to the $x$-axis is 7, then $p=$ ( ).

A. 14 B. 15 C. 16 D. 28 E. 32

12. In a shooting game, the probabilities of John and Anna hitting the target successfully in each shot are $\frac{1}{3}$ and $\frac{2}{3}$, respectively. If each of them shoots three times, the probability that the total number of successful hits by John and Anna is 4 is ( ).

A. $\frac{16}{27}$ B. $\frac{64}{81}$ C. $\frac{25}{81}$ D. $\frac{58}{243}$ E. $\frac{34}{243}$

13. Given $S_n$ be the sum of the first $n$ terms of the arithmetic sequence $\{a_n\}_{n=1}^{\infty}$, and $2S_3^2=3S_2S_4$, $a_1=4$. Then $a_7=$ ( ).

A. $-14$ B. $-8$ C. $-2$ D. 10 E. 18

14. If $x$, $y$ satisfy the constraints $\begin{cases} 3x+4y\le 7 \\ x-2y\ge -1 \\ y\ge -1 \end{cases}$, then the maximum value for $z=3x+y$ is ( ).

A. 4 B. 6 C. 7 D. 10 E. 11

15. The function $f(x)$ is defined on $\mathbb{R}$ and satisfies $f(x)=f(x+2)$. For $x\in[4,6]$, $f(x)=1+|x-5|$. The following inequality ( ) is false.

A. $f(\sin\frac{\pi}{6})>f(\cos\frac{\pi}{6})$ B. $f(\sin\frac{\pi}{3})>f(\cos\frac{\pi}{3})$ C. $f(\cos\pi)<f(\sin\pi)$

D. $f(\sin\frac{2\pi}{3})<f(\cos\frac{2\pi}{3})$ E. $f(\sin\frac{\pi}{2})<f(\cos\frac{\pi}{2})$

**<u>Part II</u> Problem-solving questions.**

1. Suppose the sum of the first $n$ terms of the sequence $\{a_n\}_{n=1}^{\infty}$ is $S_n = n^2 + 2n$. In the geometric sequence $\{b_n\}_{n=1}^{\infty}$ with positive common ratio, $b_1 = 2$ and $b_3 = 2a_4$.

   (a) Find the general terms of the sequences $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$. (4 marks)

   (b) Let $c_n = a_n b_n$. Find the sum $T_n$ of the first $n$ terms of the sequence $\{c_n\}_{n=1}^{\infty}$. (4 marks)

2. Let $f(x) = 8x^4 + ax^3 + bx^2 + cx + 9$, where $a$, $b$ and $c$ are constants. It is given that the remainder is $-10$ when $f(x)$ is divided by $x + 1$ and $f(x) \equiv (px^2 - 3x + 3)(2x^2 + qx + r)$ where $p$, $q$ and $r$ are constants.

   (a) Find the values of $p, q$ and $r$. (3 marks)

   (b) Find real roots of the equation $f(x) = 0$. (5 marks)

3. In $\triangle ABC$, $\sin(A + B) = 6\sin^2 \dfrac{C}{2}$.

   (a) Find $\cos C$. (4 marks)

   (b) If $\angle A = 45°$, find $\sin 2B$. (4 marks)

4. In $\triangle ABC$, $AB = AC$, $AD = AE$. The point $F$ lies on $AC$, $DF$ intersects $BE$ at the point $G$, and $\angle AFD = \angle DEB$.

   (a) Prove that $\triangle DEG \sim \triangle DFE$. (3 marks)

   (b) Prove that $\triangle DEF \sim \triangle BDE$. (3 marks)

   (c) Prove that $DG \cdot DF = DB \cdot EF$. (2 marks)

5. Given two points $A(-2\sqrt{2}, 0)$ and $B(2\sqrt{2}, 0)$, a moving point $M(x, y)$ satisfies that the product of the slopes of the lines $AM$ and $BM$ is $-\dfrac{1}{2}$. Let the locus of the point $M$ be the curve $\mathcal{C}$.

   (a) Find the equation of the curve $\mathcal{C}$. (4 marks)

   (b) The straight line $\ell : y = kx + b$ $(k, b \neq 0)$ intersects the curve $C$ at points $P$ and $Q$. The midpoint of the line segment $PQ$ is $D$, and the origin of coordinates is $O$. Find the slope of the straight line $OD$ (in terms of $k$). (4 marks)

# Suggested Answer

**<u>Part I</u> Multiple choice questions.**

| Question Number | Best Answer |
|---|---|
| 1 | A |
| 2 | D |
| 3 | A |
| 4 | C |
| 5 | D |
| 6 | E |
| 7 | C |
| 8 | C |
| 9 | B |
| 10 | E |
| 11 | E |
| 12 | D |
| 13 | B |
| 14 | D |
| 15 | B |

**Part II** **Problem-solving questions.**

1. (a) When $n = 1$, $a_1 = 3$. When $n \geq 2$, $a_n = S_n - S_{n-1} = n^2 + 2n - (n-1)^2 - 2(n-1) = 2n + 1$. Then $n \geq 1$, $a_n = 2n + 1$. Since $a_4 = 9$, $b_3 = 2a_4 = 18$. Let the common ratio of the geometric series $\{b_n\}_{n=1}^{\infty}$ is $q(> 0)$. Then $b_3 = b_1 q^2$. Since $b_1 = 2$, $18 = 2q^2$. Then $q = 3$. Thus $b_n = 2 \cdot 3^{n-1}$.

   (b) Since $c_n = a_n b_n$ and $T_n = c_1 + c_2 + \cdots + c_n = 3 \cdot 2 + 5 \cdot 6 + 7 \cdot 18 + \cdots + (2n+1) \cdot 2 \cdot 3^{n-1}$, we have $3T_n = 3 \cdot 6 + 5 \cdot 18 + \cdots + (2n-1) \cdot 2 \cdot 3^{n-1} + (2n+1) \cdot 2 \cdot 3^n$. Then $2T_n = 3T_n - T_n = -3 \cdot 2 - 2 \cdot 6 - 2 \cdot 18 + \cdots - 2 \cdot 2 \cdot 3^{n-1} + (2n+1) \cdot 2 \cdot 3^n = -6 - 2(6 + 18 + \cdots + 2 \cdot 3^{n-1}) + (2n+1) \cdot 2 \cdot 3^n = -6 - 2 \cdot \dfrac{6(1 - 3^{n-1})}{1-3} + (2n+1) \cdot 2 \cdot 3^n = 4n \cdot 3^n$ . Thus $T_n = 2n \cdot 3^n$.

2. (a) Considering the coefficients of $x^4$ and the constant term of $f(x)$, easily we can get $2p = 8$ and $3r = 9$. Then $p = 4$ and $r = 3$. Thus $f(x) = (4x^2 - 3x + 3)(2x^2 + qx + 3)$. Since the remainder is $-10$ when $f(x)$ is divided by $x + 1$, we have $f(-1) = -10$. Thus $10(5 - q) = -10$, which implies that $q = 6$.

   (b) If $f(x) = (4x^2 - 3x + 3)(2x^2 + 6x + 3) = 0$, then $4x^2 - 3x + 3 = 0$ or $2x^2 + 6x + 3 = 0$. For the equation $4x^2 - 3x + 3 = 0$, its determinant $\Delta = (-3)^2 - 4(4)(3) = -39 < 0$, which implies that the equation has no real solution. For the equation $2x^2 + 6x + 3 = 0$, its determinant $\Delta = 6^2 - 4(2)(3) = 12 > 0$, which implies that the equation has two different real solutions. The two solutions of the equation are $x_1 = \dfrac{-3 + \sqrt{3}}{2}$ and $x_2 = \dfrac{-3 - \sqrt{3}}{2}$.

3. (a) Since $\sin C = \sin(A+B) = 6 \sin^2 \frac{C}{2}$, $\sin C = 3(1 - \cos C)$. Using the identity $\sin^2 C + \cos^2 C = 1$, we get $9(1 - \cos C)^2 + \cos^2 C = 1$, which implies that $5\cos^2 C - 9\cos C + 4 = (5\cos C - 4)(\cos C - 1) = 0$. Because C is an interior angle of the triangle, we have $\cos C - 1 \neq 0$. Therefore, $5\cos C - 4 = 0$, which gives $\cos C = \dfrac{4}{5}$.

   (b) Since $\angle A = 45°$, we have $\sin 2A = 1$ and $\cos 2A = 0$. Then $\sin(2B) = -\sin(2A + 2C) = -\sin 2A \cos 2C - \cos 2A \sin 2C = -\cos 2C = 1 - 2\cos^2 C = 1 - (\frac{4}{5})^2 = -\dfrac{7}{25}$.

4. (a) In $\triangle DEG$ and $\triangle DFE$, $\angle DEG = \angle DFE$ (given), and $\angle EDG = \angle FDE$ (the same angle). Therefore $\triangle DEG \sim \triangle DFE$. (There are multiple ways to solve this problem, which will not be enumerated

here.)

(b) Since $AD = AE$, we have $\angle ADE = \angle AED$. Thus $\angle DEF = \angle BDE$. Since $\angle AFD = \angle DEB$, we have $\angle EFD = \angle DEB$. Thus $\triangle DEF \sim \triangle BDE$. (There are multiple ways to solve this problem, which will not be enumerated here.)

(c) By $\triangle DEG \sim \triangle DFE$, we have $\frac{DE}{DF} = \frac{DG}{DE}$. Then $DE^2 = DG \cdot DF$. By $\triangle DEF \sim \triangle BDE$, we have $\frac{DE}{BD} = \frac{EF}{DE}$. Then $DE^2 = DB \cdot EF$. Therefore $DG \cdot DF = DB \cdot EF$.

5. (a) Given $M(x, y)$, the slope of $AM$ is $k_{AM} = \dfrac{y}{x + 2\sqrt{2}}$, and the slope of $BM$is $k_{BM} = \dfrac{y}{x - 2\sqrt{2}}$. Thus, $k_{AM} \times k_{BM} = -\dfrac{1}{2}$, which is $\dfrac{y}{x + 2\sqrt{2}} \times \dfrac{y}{x - 2\sqrt{2}} = -\dfrac{1}{2}$. Simplify the equation and we can get the equation of the curve $\mathcal{C}$ $\dfrac{x^2}{8} + \dfrac{y^2}{4} = 1$.

(b) Combine the equation of the straight line $\ell$ and the curve $\mathcal{C}$, $\begin{cases} y = kx + b \\ \dfrac{x^2}{8} + \dfrac{y^2}{4} = 1 \end{cases}$. Eliminating $y$, we get $\dfrac{x^2}{8} + \dfrac{(kx + b)^2}{4} = 1$, which is $(2k^2 + 1)x^2 + 4kbx + 2b^2 - 8 = 0$. By Vieta's Theorem, the coordinates of the the midpoint $D$ is $x_D = -\dfrac{2kb}{2k^2 + 1}$ and $y_D = \dfrac{b}{2k^2 + 1}$. Thus the slope of the straight line $OD$ is $k_{OD} = \dfrac{y_D}{x_D} = -\dfrac{1}{2k}$.